Problem. Find the largest integer that divides $p^4 - 1$, for all $p > 3$.

Solution.

We factor $p^4 - 1$ as $(p-1)(p+1)(p^2+1)$. Since p is prime and greater than 3, p isn't divisible by 3. Therefore, either $p+1$ or $p-1$ is divisible by 3, since one of the three consecutive integers $p-1, p, p+1$ is divisible by 3. In general, out of three consecutive integers, exactly one is divisible by 3. Since p is odd, it isn't divisible by 2. Therefore, $p+1$ and $p-1$ are both divisible by 2. Since $p+1$ and $p-1$ are consecutive even numbers, one of them is divisible by 4, and the other just by 2. We get another factor of 2 from p^2+1: since p is odd, so is p^2—since the product of two odd numbers is odd. So, p^2+1 is even. We've shown that $(p-1)(p+1)(p^2+1)$ is twice divisible by 2, one divisible by 4, and once divisible by 3. Thus, the largest integer that divides $p^4 - 1$ for all $p > 3$ is $4 \times 2 \times 2 \times 3 = \boxed{48}$